ON THE SELECTIVITY OF A RESISTANCE CAPACITANCE NETWORK

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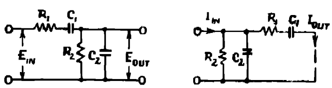
ABSTRACT. A resistance capacitance network, used in circuits for generation (Wien bridge oscillator) and measurement (Wien bridge) of low frequencies, has been analysed for the maximum selectivity condition by defining a design parameter n. It has been shown that a lower value of n gives (i) a more selective response and as such, a purer waveform in the oscillator circuit and a sharper null point in the Wien bridge circuit and (ii) a more favourable condition of operation of the active device in the oscillator circuit. The effect of cascading such networks on the selectivity of the resultant transfer characteristic has been discussed. The effect of interchanging the series and the shunt arms of the network has been considered. It has been shown that if n is high, the resulting network has a characteristic similar to that of a Wien bridge and is superior to the latter in some respects.

INTRODUCTION

The RC network shown in Fig. 1(a) is used in a vacuum tube oscillator circuit for generation and in the Wien bridge circuit for measurement of low frequencies while its current dual shown in Fig. 1(b) is used in a low frequency transistor oscillator. In such applications, it has been conventional to use $R_1 = R_2$ and $C_1 = C_2$; under these conditions, the network has a Q (Morris, 1954) equal to 0.33 only. In this paper, the effect of unequal elements on the selectivity of the transfer characteristic has been investigated. By defining a design parameter n as $n = (R_2/R_1)^{\frac{1}{2}} = (C_1/C_2)^{\frac{1}{2}}$, it has been found that the increase in Q is of the order of 50 % for very small values of n. Thus using a small n, a purer waveform can be obtained in the oscillator circuit and a sharper null point in the Wien bridge circuit. It is also found that using a small value of n ensures a better operating condition for the active device in the oscillator circuit.



(a) (b)

Fig. 1.—The RC networks under consideration. The network (b) is the current dual of the network (a).

It is known that properly cascading two selective networks having the same resonance frequency yields a response characteristic that is more selective than

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the response of either network. Thus a still better waveform can be obtained in the Wien bridge oscillator if a cascade of two or more RC networks of the form of Fig. 1 is used as the frequency selective network. The conditions and effects of proper cascading are discussed in this paper.

Finally, the networks obtained by interchanging the series and shunt arms of the networks of Fig. 1 have been studied. It has been found that by properly choosing n, these networks have a characteristic similar to that of a Wien bridge and that in some respects, they possess some advantages over the Wien bridge.

ANALYSIS OF THE RC NETWORK

Driven by an ideal voltage generator and working into an open circuited load, the network of Fig. 1(a) has a voltage transfer function given by

$$\beta = \frac{1}{1+\dfrac{C_1R_1+C_2R_2}{C_1R_2}+pC_2R_1+\dfrac{1}{pC_1R_2}} \quad \dots \quad (1)$$

where $p = j\omega$, ω being the frequency in radians/sec. The above expression also represents the current transfer function of the network of Fig. 1(b) when an ideal current generator is connected across the input terminals and the output terminals are short circuited. From (1), the resonance frequency is given by

$$\omega_0^2 = \frac{1}{C_1C_2R_1R_2} \quad \dots \quad (2)$$

Let us define a design parameter n as follows :

$$n = (R_2/R_1)^{\frac{1}{2}} = (C_1/C_2)^{\frac{1}{2}}$$

Then the components of the networks of Fig. 1 can be expressed in terms of a resistance parameter R, a capacitance parameter C and n as follows :

$$R_1 = R/n,\ R_2 = nR,\ C_1 = nC \quad \text{and} \quad C_2 = C/n \quad \dots \quad (3)$$

From (2) and (3), we have $\omega_0 = 1/(RC)$. Thus a variation of n will have no effect on ω_0. Also from (1) and (3), we have,

$$\beta = \frac{n^2u}{1+(2+n^2)u+u^2} \quad \dots \quad (4)$$

where $u = pCR$. Applying Morris' definition of Q, we have from (4),

$$Q = \frac{1}{n^2+2} \quad \dots \quad (5)$$

For the conventional circuit, $n = 1$ so that $Q = 0.33$. Expression (5) shows that Q can be increased above this value by decreasing n, a maximum value of

0.50 being reached when n tends to zero. At the resonance frequency, $u = j$ so that from (4), the resonant response is given by

$$\beta_0 = \frac{n^2}{n^2+2} \qquad \dots \quad (6)$$

At $n = 1$, $\beta_0 = 1/3$; as n decreases, β_0 also decreases and tends to zero when n tends to zero. This is not, however, very important because it only means that the gain (open loop voltage gain or the short circuit current gain according as the network of Fig. 1(a) or (b) is used) of the oscillator circuit has to be increased by the proper amount. Thus at $n = 0.30$, $\beta_0 = 0.04$; if this network is used in an oscillator, the minimum gain required for oscillations to occur is 25, a value which is not at all difficult to be realised with two stages of amplification as used in such oscillators. The improvement in Q is however as much as 45%. Fig. 2 shows the variations of Q and β_0 with n.

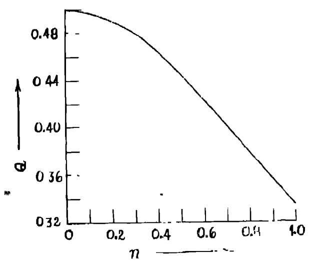

Fig. 2. (a) Showing the variation of Q with n

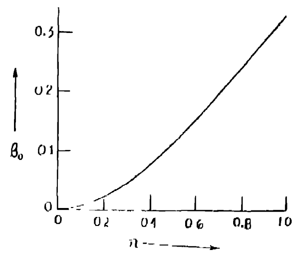

Fig. 2. (b) Showing the variation of β_0 with n

In a vacuum tube Wien bridge oscillator, the input terminals of the network of Fig. 1(a) are connected across the plate to cathode of the second valve while the output terminals are connected across the grid to cathode of the first valve of a two stage RC coupled amplifier. It is thus desirable that the output impedance of the second valve should be negligible compared with the input impedance of the network and the input impedance of the first valve should be very high compared with the output impedance of the network. Since the grid to cathode impedance of a vacuum tube is normally very high, the second condition is usually satisfied in practical circuits. But, with the conventional RC network ($n = 1$), the first condition cannot always be satisfied. This results in (i) loading of the second valve and therefore, reduction of the available gain from this stage and (ii) a deviation of the frequency from the design value $1/(RC)$. In a precision variable frequency oscillator, it is highly desirable that the frequency should be controlled by the elements of the RC network only so that the latter effect has to be annulled by the use of a compensating resistance placed between the cathodes of the two valves (Davidson, 1952).

The input impedance of the RC network of Fig. 1(a) with the elements given by equations (3), is

$$Z_{in} = R\,\frac{(u+1)^2+n^2u}{nu(u+1)} \qquad \dots \quad (7)$$

For the conventional network, $n = 1$ so that

$$Z_{i1} = R\,\frac{(u+1)^2+u}{u(u+1)}$$

Therefore,

$$r = \frac{Z_{in}}{Z_{i1}} = \frac{(u+1)^2+n^2u}{n\{(u+1)^2+u\}}$$

At the resonance frequency, $u = j$ so that

$$r_0 = \frac{n^2+2}{3n} \qquad \dots \quad (8)$$

Equation (8) shows that r_0 increases as n decreases. If $n = 0.1$ then $r_0 = 6.7$; this increased input impedance ensures a better operating condition of the second valve and a less deviation of the frequency from the value $1/(RC)$. If a sufficiently small n can be used, then the use of a compensating resistance can be avoided.

In a transistor oscillator using the network of Fig. 1(b) the network will, in general, reduce the available current gain of the second transistor and will cause a departure of the frequency from the value $1/(RC)$. This latter effect is more important as the transistor parameters vary considerably with the various d.c. voltages and with frequency. It is thus desirable that the input impedance of the network should be small compared with the output impedance of the second transistor and the output impedance of the network should be large compared with the input impedance of the first transistor.

With the elements given by equations (3), the input impedance of the network of Fig. 1(b) is

$$Z'_{in} = R\,\frac{n(u+1)}{n^2u+(u+1)^2}$$

For the conventional network,

$$Z'_{i1} = R\,\frac{(u+1)}{u+(u+1)^2}$$

so that

$$r' = \frac{Z'_{in}}{Z'_{i1}} = \frac{n\{u+(u+1)^2\}}{n^2u+(u+1)^2}$$

and at resonance,

$$r'_0 = \frac{3n}{n^2+2}$$

Thus r'_0 decreases with decreasing n and approaches zero as n tends to zero. At $n = 0.1$, r'_0 has a value 0.149. Also Z'_{n0}, the output impedance of the network of Fig. 1(b) is the same as Z_{n} given by equation (7). Thus at resonance, the ratio Z'_{on}/Z'_{o1} will be the same as r_0 given by equation (8), which increases with decreasing n.

Thus we conclude that a lower value of n gives a higher selectivity and a more favourable operation of the oscillator circuit with either vacuum tubes or transistors as the active elements. This improvement is obtained at the cost of of an increased gain of the active elements.

USE IN THE WIEN BRIDGE

Fig. 3 shows the network of Fig. 1(a) with elements given by equations (3), inserted in the two arms of a Wheatstone's bridge the other arms of which are formed by resistances whose values are so chosen that null occurs at a frequency $1/(CR)$. The transfer function of the bridge is given by

$$t = \frac{n^2}{n^2+2} - \frac{n^2}{(n^2+2)+u+1/u}$$

$$= \frac{n^2}{n^2+2} \cdot \frac{u^2+1}{u^2+(n^2+2)u+1}.$$

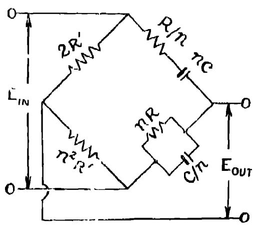

Fig. 3. The Wien bridge.

Thus Q of the network is the same as that given by equation (5). The maximum response of the network occurs at $u = 0$ and at $u = \infty$ and is given by equation

(6). Thus a small value of n gives a sharper null point, but since the maximum response of the network is reduced, the amplifier in the detector circuit has to be more sensitive or the input voltage is to be raised by the proper amount.

EFFECT OF CASCADING

For the construction of a fixed frequency oscillator, if the available gain of the amplifier is considerably greater than the required value, it will be convenient to use a cascade of two or more RC networks of the form of Fig. 1. The resulting circuit will give a better waveform than that obtained with a single network. Cascading of more than two sections will not however be practical as the output will then be heavily attenuated For a proper cascading of the networks of the form of Fig. 1(a), the output impedance of the first network should be small compared to the input impedance of the second network, while if the networks are of the form of Fig. 1(b), the reverse should be true. The following analysis shows that in a cascade of two networks of the form of Fig 1(a) or (b) with the same values of n and ω_0, the above conditions are satisfied if n is less than 0.5

The transfer function of the cascaded network is

$$\beta_2 = \frac{Z_2^2}{Z_1Z_2+(Z_1+Z_2)^2} = \frac{\beta^2}{1+\Delta}$$

where β is the transfer function of a single network given by equation (4), Z_1 and Z_2 are the impedances of the series and the shunt arms of a single network and $\Delta = Z_1Z_2/(Z_1+Z_2)^2$ is a measure of the loading of the first stage by the second. Now,

$$\Delta = \frac{Z_1Z_2}{(Z_1+Z_2)^2} = \frac{Z_1}{Z_2}\,\beta^2 = \frac{(u+1)^2}{n^2u}\,\beta^2$$

At a frequency given by $u = j$,

$$\Delta = \frac{2n^2}{(n^2+2)^2}$$

At $n = 0.5$, $\Delta = 0.099$ so that for $n < 0.5$, $\beta_2 \simeq \beta^2$. Combining this with (4) gives

$$\beta_2 \simeq \frac{\{n^2/(n^2+2)\}^2}{\{1+j(x-1/x)/(2+n^2)\}^2} \quad \ldots \quad (9)$$

where $x = \omega CR = \omega/\omega_0$ is the normalised frequency. The response at resonance is given by $\beta_{20} = \{n^2/(n^2+2)\}^2$. Morris' definition is not applicable here. However, for a resonance curve the definition of Morris gives the same value of Q as that obtained from the conventional definition, viz., $Q = \omega_0/(\omega_1 \sim \omega_2) = 1/(x_1 \sim x_2)$, where ω_1 and ω_2 are the frequencies at which the response is 70.7% of that at resonance. Applying this definition to (9), it can be shown that Q of the cascaded network is given by

$$Q_2 = \frac{1.53}{n^2+2} = 1.53Q$$

If $n = 0.1$, then $Q_2 = 0.76$ which is nearly equal to its maximum value 0.765.

EFFECT OF INTERCHANGING THE ARMS

If the series and the shunt arms of the networks of Fig. 1 are interchanged, the transfer function of the resulting network will be given by

$$\beta' = \frac{Z_1}{Z_1+Z_2} = \frac{u^2+2u+1}{u^2+(2+n^2)u+1}$$

Thus again, $Q = 1/(2+n^2)$. In terms of the normalised frequency,

$$\beta' = \frac{(1-x^2)+2jx}{(1-x^2)+(2+n^2)jx}$$

This shows that β' has a maximum value of unity at both $x = 0$ and $x = \infty$ and a minimum value of $2/(2+n^2)$ at $x = 1$. Thus the network characteristic is similar to that of a Wien bridge excepting that the minimum response is not zero. β' can be made to approach zero by using a large value of n, but then the selectivity will be poor. If a compromise is made between the two, then the network can be used for measurement of low frequencies. With a high impedance detector (e.g. a vacuum tube amplifier-rectifier arrangement), the network of Fig. 1(a) with interchanged arms will be suitable for measuring the frequency of a low impedance source (e.g. a vacuum tube oscillator) With a low impedance detector (e.g. a transistor amplifier-rectifier arrangement), the network of Fig. 1(b) with interchanged arms will be suitable for measuring the frequency of a high impedance source (e.g. a transistor oscillator). In this application, the networks under consideration have the advantages over a Wien bridge of (i) requiring a less number of components, (ii) possessing a common input and output terminal thus avoiding the necessity of using a balance to unbalance transformer, and (iii) a simpler layout.

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